Time-Variant Strength Capacity Model for GFRP Bars Embedded in Concrete

Paolo Gardoni¹; David Trejo, A.M.ASCE²; and Young Hoon Kim, A.M.ASCE³

Abstract: Glass fiber-reinforced polymer (GFRP) concrete reinforcement exhibits high strength, is lightweight, can decrease time of construction, and is corrosion resistant. However, research has shown that chemical reactions deteriorate the GFRP reinforcing bars over time, resulting in a reduced tensile capacity. This paper develops a time-variant probabilistic model to predict the tensile capacity of GFRP bars embedded in concrete. The developed model is probabilistic to properly account for the relevant sources of uncertainties, including the statistical uncertainty in the estimation of the unknown model parameters (because of the finite sample size), the model error associated with the inexact model form (e.g., a linear expression is used when the actual and unknown relations are nonlinear), and missing variables (i.e., the model only includes a subset of the variables that influence the quantity of interest.) The proposed model is based on a general diffusion model, in which water or ions penetrate the GFRP bar matrix and degrade the glass fiber-resin interface. The model indicates that GFRP reinforcement bars with larger diameters exhibit lower rates of capacity loss. The proposed probabilistic model is used to assess the probability of not meeting the tensile strength requirement based on specifications over time and can be used to assess the safety and performance of GFRP reinforced systems. Sensitivity and importance analyses are carried out to explore the effect of the parameters and random variables on the probability estimates.

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Introduction

Glass fiber-reinforced polymer (GFRP) reinforcement is being used as reinforcement for concrete structures. Significant research has been performed on this material (Vijay and Gangarao 1999; Dejke 2001; Micelli et al. 2001; Almusallam et al. 2002; Giemacky et al. 2002; Svecova et al. 2002; Micelli and Nanni 2004; Mukherjee and Arwikar 2005; Bakis et al. 2005; Mufti et al. 2007a,b; Cain 2008; Huang and Abouataha 2010). GFRP reinforcement has many advantages over conventional steel reinforcement; it is lightweight, has a high strength-to-weight ratio, and the chemical or electrochemical reactions between the environment and the GFRP do not result in products with increased volumes that might result in concrete cracking and spalling (Bradberry 2001). However, Trejo et al. (2005; 2011) have evaluated the potential use of GFRP reinforcement for bridge decks and reported experimental results that indicated tensile strength loss over time. In particular, Trejo et al. (2011) reported experimental tension test data for 160 GFRP reinforcement bars embedded in concrete and exposed to actual environmental conditions [mean annual temperature of 23°C (73°F) and average annual precipitation of rain of 1.008 mm (39.7 in.)] for 7 years. The experimental program included the assessment of two GFRP bar sizes, 16 and 19 mm (5/8 and 3/4 in.) from three manufacturers. Although 7 years of exposure is shorter than the typical service time of concrete structures, it is the longest exposure time for which data are currently available.

This paper develops a state-of-the-art, time-variant deterioration model for GFRP reinforcing bars embedded in concrete. Unknown model parameters are calibrated using experimental data from Trejo et al. (2011) and 27 additional data available in the literature on GFRP reinforcement embedded in concrete (Tannous and Saadatmanesh 1998; Dejke 2001). The model is developed based on general diffusion transport principles and predicts the residual capacity of GFRP bars embedded in concrete. The developed model is probabilistic and properly accounts for the relevant sources of information, including the principles of diffusion transport and experimental data. The model also accounts for uncertainties, including the statistical uncertainty in the estimation of the unknown model parameters and the model error associated with the inexact model form (e.g., a linear expression is used when the actual and unknown relations are nonlinear) and missing variables (i.e., the model only includes a subset of the variables that influence the quantity of interest.) The model shows that the residual tensile strength capacity is a function of time of embedment in concrete, $t$, the diffusion coefficient of the GFRP polymer matrix, $D_T$, and the GFRP bar radius, $r$. In particular, the model predicts that GFRP reinforcement with smaller diameters exhibit faster reductions in residual tensile strength than larger diameter GFRP reinforcing bars.

The developed time-variant model provides the required information to assess the safety and performance of GFRP reinforcing bars embedded in decks, pavements, and other infrastructure elements over time. In this paper, the proposed probabilistic model is used to assess the probability of not meeting the tensile strength requirement over time and at different ambient temperatures, $T$. The...
first-order reliability method (FORM) and Monte Carlo (MC) simulations are used to assess estimates of this probability, along with confidence bounds, to implicitly or explicitly reflect the influence of the epistemic uncertainties. Sensitivity and importance analyses are carried out to explore the effect of changes in the parameters and random variables on the probability estimates. Based on the results of the importance analysis, an approximate closed form of the probability of not meeting the tensile strength requirement is also proposed.

This paper first discusses the deterministic models currently available in the literature to predict the tensile strength capacity of GFRP bars. Next, the minimum design requirements according to American Concrete Institute (ACI) Committee 440.1R (ACI 2007) are reviewed. The following section formulates the probabilistic capacity model, which is then calibrated in the following section using experimental data. The probability of not meeting the ACI design requirement is then estimated and the sensitivity and importance analyses are carried out. Finally, the approximate closed form to estimate the probability of not meeting the design requirements is proposed.

Models from the Literature

The constituent materials of GFRP bars and their exposure conditions play a significant role in the performance of GFRP bars. Deterministic models have been developed to predict the residual strengths at different times, thereby providing the designer with possible estimates of bar capacity at later ages. These residual strengths, or factored residual strengths, could then be used in the design of GFRP RC elements. This section provides a review of the available models for assessing the residual strength of GFRP bars embedded in concrete. Two limitations common to all the available models are that (1) only limited long-term data on the performance of GFRP bars embedded in concrete were used (most data are for accelerated exposure conditions, and the exposure periods were less than three years), and (2) the models are typically biased and deterministic (i.e., they tend to be implicitly conservative and do not account for the uncertainties inherent in the problem), and therefore, they provide limited value for assessing the reliability or safety of concrete structures reinforced with GFRP bars.

The rate of diffusion of elements or compounds (e.g., water, alkalis, and hydroxyl ions) has a significant impact on the residual strength of GFRP bars. Simply, the compounds are transported to the glass fibers where they react with the glass and debond the glass fibers from the resin composite. This results in a reduced residual strength of GFRP bars, particularly, the tension. Therefore, Katsuki and Uomoto (1995) proposed a predictive model based on Fick’s first law. The authors assumed that the tensile strength of GFRP bars can be determined quantitatively by the amount of alkali penetration area into the bars and recommended that the depth of penetration can be calculated using the following equation:

\[ X = \sqrt{2 \cdot D_T \cdot C \cdot t} \]  

(1)

where \( X \) = depth of penetration from the surface; \( D_T \) = diffusion coefficient at temperature \( T \); \( C \) = alkali concentration (percent); and \( t \) is the curing time. Various units can be used in this equation, and the units of the square root of the product should result in a length unit.

Katsuki and Uomoto (1995) assumed that, as the glass fibers are exposed to a diffusing solution, the fibers exhibit complete failure and no longer contribute to the bars’ load bearing capacity. The authors then proposed the following equation for estimating the residual strength:

\[ \sigma_f = \left( 1 - \sqrt{\frac{2}{r} \cdot D_T \cdot C \cdot t} \right)^2 \cdot \sigma_0 \]  

(2)

where \( \sigma_0 \) and \( \sigma_f \) = tensile strengths before and after exposure (stress units), respectively. A similar approach was proposed by Tannous and Saadatmanesh (1998).

Trejo et al. (2005) reported that the assumption of complete loss of glass fiber capacity upon immediate contact with the solution likely overestimated the loss of capacity and proposed an exposure factor, \( \lambda \), to account for this time-dependent deterioration of the bond between the glass and resin. The formula, modified with the exposure factor, is as follows:

\[ \sigma_f = \left( 1 - \sqrt{\frac{2}{r} \cdot D_T \cdot \lambda \cdot t} \right)^2 \cdot \sigma_0 \]  

(3)

where the definitions of the variables have already been reported, and \( \lambda \) is a model parameter that typically varies with the solution type and exposure time.

Because the rate of chemical reactions is dependent on temperature, it would be beneficial to consider the influence of temperature on tensile strength capacity. The Arrhenius equation has been widely used to establish relationships between degradation data from laboratory-accelerated aging tests and service life of field structures. Proctor et al. (1982) suggested that time–temperature relationships can be obtained using deterioration data of material after exposure in concrete at different temperatures. If the shape of the residual strength curves is a function of the logarithm of time and is similar for different temperatures, the Arrhenius equation could be applicable. Katsuki and Uomoto (1995) reported that the Arrhenius equation offers a good correlation between the temperature and the rate of diffusivity and chemical reactions. The Arrhenius equation is shown as follows:

\[ k_T = A e^{-E_a/RT} \]  

(4)

where \( k_T \) = the rate constant; \( A \) = the frequency factor; \( E_a \) = the activation energy (kilojoules); \( R \) = the universal gas constant (kilojoules per molar \( \times \) kelvins); and \( T \) = the absolute temperature (kelvins).

Eq. (4) can also be used to determine the influence of temperature on the diffusion coefficient, \( D_T \), by substituting \( k_T \) with \( D_T \) as follows:

\[ D_T = A e^{-E_a/RT} \]  

(5)

Eq. (5) can then be used to determine the relative change in diffusion rates as follows:

\[ \frac{D_T}{D_{T, ref}} = \frac{A \cdot e^{-E_a/RT}}{A \cdot e^{-E_a/R_{T, ref}}} = e^{E_a/\left(R \left( 1/T_{ref} - 1/T \right) \right)} \]  

(6)

where \( T \) and \( T_{ref} \) = exposure temperatures (in kelvins). Assuming data on capacity loss can be generated at some reference temperature, \( T_{ref} \), an estimate on the capacity loss of GFRP bars at a different temperature can be determined by including Eq. (6) into Eq. (3) as follows:

\[ \sigma_f = \left( 1 - \sqrt{\frac{2 \cdot D_{T, ref} \cdot e^{E_a/\left(R \left( 1/T_{ref} - 1/T \right) \right)} \cdot \lambda \cdot t}{r}} \right)^2 \cdot \sigma_0 \]  

(7)

Note that at \( T = T_{ref} \), the exponential term goes to unity, and Eqs. (3) and (7) are the same. Therefore, by determining the capacity loss of
GFRP bars at a reference temperature, information can be generated on the performance of GFRP bars at other temperatures. Accelerated short-term tests indicate that the activation energy is significantly influenced by the chemical composition of the GFRP bar and pore solution of the concrete (Chen et al. 2006). In addition, short-term accelerated tests can lead to high uncertainty of the activation energy value for the GFRP bars exposed to high pH pore solutions (Chin et al. 2001; Chen et al. 2006). Yu et al. (2007) also reported that the determination of parameters in the Arrhenius law were also highly uncertain. Because determining the exact value of these parameters is beyond the scope of this paper, the activation energy is assumed to be the average value of Proctor’s extensive data set (range, 89–93 kJ/mol). The data set is not from the GFRP bars embedded in concrete, but the data set is considered to be reasonable for the degradation of glass fibers in cementitious materials for long-term exposure.

The literature indicates that the loss of GFRP bar capacity is a result of water or alkaline solution diffusing through the polymer to the glass fiber–polymer interface, resulting in deterioration of the glass fiber, which results in debonding of the glass fiber from the polymer. As the water or solution continues to penetrate the polymer, more glass fibers are debonded, resulting in further reduction in capacity.

**Tensile Strength Design Requirements**

ACI Committee 440.1R (ACI 2007) and AASHTO Load and Resistance Factor Design (AASHTO 2008) specifications require use of an environmental reduction factor as a design parameter when considering the reduction in tensile strength of GFRP in actual structures. This reduction factor, \( C_E \), depends on the exposure conditions of the GFRP RC. For concrete not exposed to earth and weather, the reduction factor is 0.8, and for concrete exposed to earth and weather, the reduction factor is 0.7. The design tensile strength, \( f_{fu} \), of GFRP reinforcing bar considering these required reductions can then be determined as follows:

\[
f_{fu} = C_E f_{fu}^e
\]  

where \( f_{fu}^e \) is guaranteed ultimate tensile strength (GUTS) of a GFRP bar; and \( C_E \) = environmental reduction factor equal to 0.8 for GFRP embedded in concrete not exposed to earth and weather and equal to 0.7 for GFRP RC exposed to earth and weather. The GUTS is defined as the mean tensile strength of a set of test specimens minus three SDs (\( f_{fu} = f_{fu,ave} - 3\sigma \)).

**Formulation of the Proposed Probabilistic Capacity Model**

By substituting Eq. (6) in Eq. (3) and replacing the square root with an unknown parameter \( \alpha \), the model in Eq. (3) based on Trejo et al. (2005) is modified into the following phenomenologic model:

\[
\sigma_t(x, \Theta) = \left(1 - \lambda \left[\frac{D_{T,ref} \cdot e^{E_u/R(1/T_{ref} - 1/T)} \cdot t}{t^2}\right]^{\alpha}\right) \cdot \sigma_0
\]  

where each term has the same definition as in Eq. (3), and \( \alpha \) and \( \lambda \) are two unknown model parameters used to fit the data. To account for the uncertainty in \( \sigma_0 \) and \( \sigma_t \), Eq. (9) is then modified as follows by adding some variability in \( \sigma_0 \) and in the effects of the deterioration:

\[
\sigma_t = \left\{1 - \lambda \left[\frac{D_{T,ref} \cdot e^{E_u/R(1/T_{ref} - 1/T)} \cdot t}{t^2}\right]^{\alpha}\right\} \cdot \sigma_0
\]  

where \( x = (D_{T,ref}, E_u, R, t, T) \) is a vector of basic variables (i.e., material properties, geometry, and temperature); \( \sigma_0 \) = an error term that captures the variability of \( \sigma_0 \) around its mean \( \mu_{\sigma_0} \); \( s \cdot \varepsilon \) = an error term that captures the variability in the reduction term \( \lambda \left[\frac{D_{T,ref} \cdot e^{E_u/R(1/T_{ref} - 1/T)} \cdot t}{t^2}\right]^{\alpha}\); \( \varepsilon_0 \) and \( \varepsilon \) = statistically independent identically distributed random variables with zero mean and unit variance; \( \sigma_0 \) and \( s \) = SDs of the two error terms; and \( \Theta = (\lambda, \alpha, \varepsilon_0, s) \) is a vector of unknown model parameters. Two assumptions are made in formulating the model: (1) \( \varepsilon_0 \) and \( s \) are not functions of \( x \) (homoskedasticity assumption), and (2) \( \varepsilon_0 \) and \( \varepsilon \) have a normal distribution (normality assumption). Diagnostic plots of the data and the residuals against model predictions (Rao and Tou-Tenburg 1997) were used to verify the validity of these assumptions. The specific values of the parameters in \( \Theta \) depend on the variability of the experimental data.

Based on the normality assumption, Eq. (10) can be rewritten as (Ang and Tang 2007)

\[
\sigma_t(x, \Theta) = \frac{1 - \lambda \left[\frac{D_{T,ref} \cdot e^{E_u/R(1/T_{ref} - 1/T)} \cdot t}{t^2}\right]^{\alpha}}{\sqrt{s^2 + \lambda^2 \left[\frac{D_{T,ref} \cdot e^{E_u/R(1/T_{ref} - 1/T)} \cdot t}{t^2}\right]^{2\alpha}}} \cdot \mu_{\sigma_0} + \varepsilon
\]  

Eq. (11) is used in the next section to assess the model parameters \( \Theta \) using experimental data. Later, Eq. (11) is used to develop an approximate estimate of the probability that the tensile strength requirement based on specifications is not met as a function of \( t, r, \) and \( T \).

**Model Assessment**

The model in Eq. (11) is assessed using experimental data from GFRP bars embedded in concrete for 7 years. A Bayesian approach is used to estimate the statistics (means, variances, and correlation coefficients) of the unknown parameters (Box and Tiao 1991). A Bayesian approach is particularly suitable because it allows updating of the statistics of the unknown model parameters as new data become available.

**Experimental Data**

Trejo et al. (2011) provided experimental data on the residual tensile strength capacity of 160 GFRP bars with diameters of 16 and 19 mm (5/8 and 3/4 in.) provided by three different manufacturers. The bars contained approximately 70% of unidirectional glass fibers by volume; the remaining volumes were resin, filler, and air voids. Three different bar types, P, V1, and V2, representing three manufacturers, were evaluated. It should be noted that the P bars are no longer being produced. Table 1 provides a breakdown of the GFRP specimens by bar type and size.

Schaefer (2002) reported that bar Type P was made with a polyethylene terephthalate (PET) polyester matrix and E-glass fibers.
Bar Type V1 contained E-glass fibers embedded in a vinyl ester resin. This bar was made with external helical fiber wrapping, and the surface of the bar was coated with fine sand. Bar Type V2 was composed of E-glass fibers embedded in a vinyl ester resin and had a circular cross section coated with coarser sand. Trejo et al. (2005) measured and reported a range of diffusion coefficients for the different GFRP bars evaluated at 23°C (73°F) to be from 2.88 × 10⁻¹³ m²/s (4.47 × 10⁻¹⁰ in.²/s) to 1.54 × 10⁻¹² m²/s (2.39 × 10⁻⁹ in.²/s), with a mean value of 8.90 × 10⁻¹² m²/s (1.38 × 10⁻⁹ in.²/s) and a standard deviation of 3.52 × 10⁻¹² m²/s (0.54 × 10⁻⁹ in.²/s). The statistics of the diffusion coefficients are used for the modeling in this paper.

GFRP RC samples were exposed for 7 years to outdoor environmental conditions with a mean annual temperature of 23°C (73°F) and a mean annual precipitation of 1,008 mm (39.7 in.), fairly evenly distributed throughout the year. The minimum and maximum average daily temperatures were 5°C (41°F) and 40°C (104°F), respectively. In addition, bars not exposed to outdoor environmental conditions are used in this paper as control specimens. Additional details on the material characterization of these GFRP bars can be found in Trejo et al. (2011). Although 7 years of exposure is shorter than the typical service life of concrete structures, it is the longest exposure time for which data are currently available.

Additional data on GFRP bar capacity were provided by Tannous and Saadatmanesh (1998) and Dejke (2001). Tannous and Saadatmanesh (1998) provided experimental data obtained from tension tests of 10 GFRP bars embedded in concrete for 1 and 2 years. The GFRP specimens manufactured by Kodiak FRP Rebar (Dayton, Texas) and International Grating (Houston, Texas) were constructed from E-glass fibers, had a fiber volume fraction of 72%, and consisted of polyester and vinylester resin matrices. The bar size was 10 mm (3/8 in.).

Dejke (2001) reported the residual tensile properties of 17 GFRP bars of different diameters, 9 mm (0.354 in.) and 8 mm (0.315 in.), manufactured by Hughes Brothers (Seward, Nebraska) and Fiberkonst (Malmö, Sweden), respectively. GFRP specimens were embedded in concrete under different temperature conditions, 20°C (68°F) and 40°C (104°F), for approximately 1 year. The concrete specimens with GFRP bars embedded were stored outdoors at 100% humidity to simulate a real concrete structure under highly moist conditions. After the bars were extracted from the concrete, tension tests were performed to assess the residual bar capacity.

### Bayesian Parameter Estimation

The unknown parameters $\Theta$ are estimated using the following Bayesian updating rule (Box and Tiao 1991), which updates the state of knowledge about $\Theta$ available before the data are collected into a new state of knowledge about $\Theta$, which reflects both what was known before the data are collected and the information from the data $D$.

$$ f(\Theta) = \psi(D|\Theta) \mu(\Theta) $$  \hfill (12)
Fig. 1 shows a comparison between the predicted and measured normalized tensile strength $\sigma_f/\mu_{\sigma_0}$ over time. The experimental data are shown as dots (•) for all bar types. The mean prediction ($E_0 = e = 0$) is shown as a solid line [in the top plot for the 10-mm (3/8 in.) bars, in the center plot for the 16-mm (5/8 in.) bars, and in the bottom plot for the 19-mm (3/4 in.) bars]. The dashed lines delimit the region within one SD of the mean. In addition, the horizontal dotted line represents the ACI 440 minimum capacity requirement, $f_{n0}/\mu_{\sigma_0} = 0.595$, where with reference to Eq. (8), $f_{n0} = C_E f_{u,ave} = C_E (f_{u,ave} - 3\sigma)$. An analysis of the data from the experimental program and the additional data available from the literature showed that $\sigma/\mu_{\sigma_0}$ varies between 0.02 and 0.09. For the purpose of the analysis conducted in this section, the ACI requirement was computed using $\sigma/\mu_{\sigma_0} = 0.05$. For each set of values of the input variables, the model predicts only one value of the quantity of interest [i.e., $E(\sigma/\mu_{\sigma_0})$]. In contrast, for the same set of input variables, multiple experimental outcomes are recorded. Such differences in the recorded outcomes can be attributed to differences in other influencing variables that are not measured and are not included in the proposed model. The proposed probabilistic model tells us what the expected value of the quantity of interest is for the given inputs and its variability around such an expected value to reflect the fact that there is variability in the data. It can also be observed that the decay of the mean normalized tensile capacity, $E(\sigma/\mu_{\sigma_0})$, is rapid over the first few years and gradually slows down as time increases. Furthermore, the decay is more pronounced for smaller bars than for larger bars. Fig. 2 shows the values of $t$ and $r$ for which $E(\sigma/\mu_{\sigma_0}) = C_E (f_{u,ave} - k\sigma)/\mu_{\sigma_0}$ for $k = 3, \ldots, 6$ ($k = 3$ is the value specified by ACI 440) at $T_{ref}$. For small GFRP bar sizes, $E(\sigma/\mu_{\sigma_0}) = f_{n0}/\mu_{\sigma_0}$ at a time less than the typical service life of a structure. For small GFRP bar sizes, a larger value of $k$ may be required so that $E(\sigma/\mu_{\sigma_0}) > C_E (f_{u,ave} - k\sigma)/\mu_{\sigma_0}$ during the service time.

### Probability of Not Meeting Design Specifications over Time

Following the conventional notation in reliability theory (Ditlevsen and Madsen 1996), a limit state function $g(\cdot)$ is introduced such that the event $[g(\cdot) \leq 0]$ denotes not meeting a specified capacity requirement. In particular, the ACI 440 minimum capacity requirement, $f_{n0}$, is considered.

Using the probabilistic model described in Eq. (11), a limit state function is written as

$$g(f_{n0}, x, \Theta) = \sigma_f(x, \Theta) - f_{n0} \quad (16)$$

Therefore, the probability of not meeting the design specifications at any time $t$ is written as

$$P(\Theta) = P(g(f_{n0}, x, \Theta) \leq 0 | \Theta) \quad (17)$$

where $P(\cdot | \cdot)$ indicates the conditional probability of $g(f_{n0}, x, \Theta) \leq 0$ for given values of $\Theta$. 

![Fig. 1. Comparison between the predicted and measured normalized stress $\sigma_f/\mu_{\sigma_0}$ over time](image1)

![Fig. 2. Values of $t$ and $r$ for which $E(\sigma/\mu_{\sigma_0}) = C_E (f_{u,ave} - k\sigma)/\mu_{\sigma_0}$ for $k = 3, \ldots, 6$](image2)
normality assumption already discussed, adversely, at the same increases. 
not meeting the design requirements increases with \( t \). Table 3 lists the distributions and the corresponding statistics of random variables (non-negative variables are assumed to follow a lognormal distribution), and the means and SDs are as provided in the references listed in Table 3.

### Predictive Estimate

Following Gardoni et al. (2002), the predictive estimate of Eq. (17), \( \bar{P} \), is defined as the expected value of \( P(\Theta) \) over the distribution of \( \Theta, f(\Theta) \), that is

\[
\bar{P} = \int_{\Theta} P(\Theta) f(\Theta) d\Theta \quad (18)
\]

By integrating over \( \Theta \), the epistemic uncertainties are incorporated into the predictive estimates of the fragility in an average sense. Furthermore, the corresponding generalized reliability index (Ditlevsen and Madsen 1996) is obtained as

\[
\tilde{\beta} = \Phi^{-1}(1 - \bar{P}) \quad (19)
\]

where \( \Phi^{-1}(\cdot) \) denotes the inverse of the standard normal cumulative distribution function.

The probability of not meeting the ACI 440 design requirements and the corresponding reliability index are functions of the initial radius of a GFRP bar, \( r \), (or its mean \( \bar{r} \)) and time, \( t \). Fig. 3 shows a conceptual three-dimensional plot of the probability of not meeting the design requirements as a function of the radius of a GFRP bar, \( r \), and \( t \). Consistent with the observations made for Fig. 1, it can be seen that for a specified bar size, the probability of not meeting the design requirements increases with \( t \). Conversely, at the same \( t \) the probability decreases as the bar size increases.

An analytical solution of Eq. (18) is typically not possible because the analytical form for the integral is usually not available. Fig. 4 shows predictive estimates as a function of time computed using MC simulations (Ditlevsen and Madsen 1996). The dotted line shows the probability for the 10-mm (3/8 in.) bars, the dashed line shows the probability for 16-mm (5/8 in.) bars, and the solid line shows the probability for 19-mm (3/4 in.) bars. Consistent with the observations made for Fig. 1, in 75 years, 10-mm (3/8 in.) bars reach a 0.37 probability of not meeting the ACI 440 requirements, 16-mm (5/8 in.) bars reach a 0.23 probability, and 19-mm (3/4 in.) bars reach a 0.18 probability. Note that the exposure temperature is assumed to be 23°C (73°F) in all figures, except for Fig. 5.

Fig. 5 shows the effect of exposure temperature on the probabilities estimated for 16-mm (5/8 in.) bars. The dotted line shows the probability for the temperature of 33°C (91°F), the dashed line shows the probability for the temperature of 23°C (73°F), and the solid line shows the probability for the temperature of 13°C (55°F). As the exposure temperature increases, the probability also increases. In 75 years, at the exposure temperature of 13°C (55°F), 16-mm (5/8 in.) bars reach a 0.09 probability, and at the exposure temperature of 33°C (91°F), 16-mm (5/8 in.) bars reach a 0.37 probability, which is 65% higher than a probability at the exposure temperature of 23°C (73°F).

Fig. 6 shows the \( P[g(f_{fu}, x, \Theta) = 0] \) as a function of \( r \). It can be seen that \( P[g(f_{fu}, x, \Theta) = 0] \) decreases as \( r \) increases and that the effect of \( r \) is more pronounced shortly after the bars are embedded in concrete than at later times. Finally, Fig. 7 shows a contour plot of the isoprobability lines for \( P[g(f_{fu}, x, \Theta) = 0] \) as a function of \( t \) and \( r \). The isoprobability lines connect pairs of values of \( t \) and \( r \) that correspond to the same \( P[g(f_{fu}, x, \Theta) = 0] \). Consistent with what was observed in Figs. 4 and 6, Fig. 7 shows that \( P[g(f_{fu}, x, \Theta) = 0] \) increases as \( t \) increases, and that bars with larger \( r \) are less prone to deteriorate than bars with smaller \( r \).

Fig. 8 shows a comparison between \( \bar{P} \) for 16-mm (5/8 in.) bars estimated using MC simulations (shown as a line with open dots) and using the FORM (Ditlevsen and Madsen 1996) (shown as a solid line). Although results from MC simulations are in general more accurate than the ones from FORM, the latter also provides the additional sensitivity and importance measures described in the next section. The validity of the sensitivity and importance measures is supported by the match between the FORM and MC estimates. Similar considerations can be made for 10- and 19-mm (3/8 and 3/4 in.) bars.

### Table 3. Distributions and Corresponding Statistics of Random Variables

<table>
<thead>
<tr>
<th>Random variable(a)</th>
<th>Bar size</th>
<th>Distribution</th>
<th>Mean</th>
<th>SD</th>
<th>Reference/source</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_{fu}/\mu_{ne} )</td>
<td>10 mm (3/8 in.)</td>
<td>Lognormal</td>
<td>0.595</td>
<td>0.060(b)</td>
<td>Trejo et al. (2011)</td>
</tr>
<tr>
<td>( D_{T,ref}, \text{m}^2/\text{s} ) (in(^2/\text{s}))</td>
<td>10 mm (3/8 in.)</td>
<td>Lognormal</td>
<td>(8.903 \times 10^{-13})</td>
<td>(3.522 \times 10^{-13})</td>
<td>Trejo et al. (2005)</td>
</tr>
<tr>
<td>( r, \text{mm (in.)} )</td>
<td>Lognormal</td>
<td>16 mm (5/8 in.)</td>
<td>4.5 (0.177)</td>
<td>0.56 (0.022)</td>
<td>Kulkarni (2006)</td>
</tr>
<tr>
<td>19 mm (3/4 in.)</td>
<td>7.92 (0.312)</td>
<td>0.97 (0.038)</td>
<td>Kulkarni (2006)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( r, \varepsilon_0 )</td>
<td>Lognormal</td>
<td>0.375 (9.53)</td>
<td>1.14 (0.045)</td>
<td>Kulkarni (2006)</td>
<td></td>
</tr>
</tbody>
</table>

\(a\)Random variables are assumed to be statistically independent.

\(b\)Computed assuming a 10% coefficient of variation.

### Fig. 3. Conceptual plot of the probability of not meeting ACI 440 requirements as a function of time and bar size.
An approximate estimate of $\tilde{P}$ that does not require any specialized reliability software and has no significant loss of accuracy can be obtained by first-order approximation of the limit state function in Eq. (16) around the means of the random variables. After defining the column vector of all random variables, $y = (f_{bu}, x, \Theta)$, the first-order approximation of $g(y)$ can be written as

$$g_1(y) = g(y) + \nabla_y g^T (y - \hat{y}) \tag{20}$$

where $\hat{y} = \text{point estimate (i.e., the mean)}$ of $y$; and $\nabla_y g^T = \text{transpose of the gradient of } g(y) \text{ computed at } \hat{y}$. The mean, $\mu_{s_1}$, and SD, $s_{s_1}$, of $g_1(y)$ can then be written as

$$\mu_{s_1} = g(\hat{y}) \tag{21}$$

$$s_{s_1} = \nabla_y g^T \Sigma_{yy} \nabla_y g$$

where $\Sigma_{yy} = \text{covariance matrix of } y$. The first-order approximation of $P$ can be written as

$$\tilde{P}_1 = P[g_1(f_{bu}, x, \Theta) \leq 0] = \Phi \left( \frac{\mu_{s_1}}{s_{s_1}} \right) \tag{22}$$

where $\Phi(\cdot)$ denotes the standard normal cumulative distribution function.

Fig. 8 shows $\tilde{P}_1$ for 16-mm (5/8 in.) bars (dashed line) compared with the results from MC simulations and FORM. The proposed first-order approximation is more accurate than FORM after approximately 50 years of exposure. However, the accuracy of the proposed first-order approximation is marginally lower than FORM from 0 to 40 years of exposure. Similar considerations can be made for 10- and 19-mm (3/8 and 3/4 in.) bars.

Sensitivity and Importance Analyses

The sensitivity analysis for the parameters in Eq. (16) is shown here. The importance measures of all random variables will then be presented. Based on the results of the importance analysis, an approximate closed form for the estimation of $P[g(f_{bu}, x, \Theta) \leq 0]$ is then proposed. The proposed closed-form estimates simplify the reliability analysis in engineering practice.

Sensitivity Analysis

Sensitivity analysis is used to determine to which parameter(s) the reliability of a structural component is most susceptible (Hohenbichler and Rackwitz 1986). Let $f(x, \Theta)$ be the probability density
function of the basic random variables in \( \mathbf{x} \), where \( \mathbf{\Theta} \) is a vector of distribution parameters (means, SDs, correlation coefficients, or other parameters defining the distributions of basic random variables in \( \mathbf{x} \)). The solution of the reliability problem in Eq. (17) also depends on the value of \( \mathbf{\Theta} \). In this application, the mean values of the basic random variables are considered the parameters for the sensitivity analysis. To compare the sensitivity measures of all parameters, the vector \( \mathbf{\delta} \) is computed as follows:

\[
\mathbf{\delta} = \mathbf{\alpha} \cdot \nabla_{\mathbf{\Theta}, \mathbf{\beta}}
\]

where \( \mathbf{\alpha} \) = diagonal matrix with diagonal elements given by the SD of each random variable in \( \mathbf{x} \), \( \nabla_{\mathbf{\Theta}, \mathbf{\beta}} = \) the gradient vector of \( \mathbf{\beta} \) with respect to \( \mathbf{\Theta} \) computed at the mean point, and \( \mathbf{\beta} = \) reliability index from FORM analysis. The gradient vector \( \nabla_{\mathbf{\Theta}, \mathbf{\beta}} \) is also computed by FORM analysis.

Fig. 9 shows the sensitivity measures as a function of time for 16-mm (5/8 in.) bars. Based on the formulation in Eq. (11). Similar observations can be made for the sensitivity measures for 10- and 19-mm (3/8 and 3/4 in.) bars.

**Importance Analysis**

Each random variable in Eq. (16) has a different contribution to the variability of the limit state function. Important random variables have larger effects on the variability of the limit state function than less important random variables. By considering only the uncertainties from the important random variables, the reliability problem can be simplified for engineering practice. In this paper, an importance analysis is conducted to obtain the vector of importance measures, \( \mathbf{\gamma} \), defined by Der Kiureghian and Ke (1985) as

\[
\mathbf{\gamma} = \frac{\mathbf{\alpha}^T \cdot \mathbf{J}_{\mathbf{x}} \cdot \mathbf{x} \cdot \mathbf{SD}}{\| \mathbf{\alpha}^T \cdot \mathbf{J}_{\mathbf{x}} \cdot \mathbf{x} \cdot \mathbf{SD} \|}
\]

where \( \mathbf{\alpha} = \) a row vector of the negative normalized gradient of the limit state function evaluated at the design point in standard normal space; \( \mathbf{J}_{\mathbf{x}} \) = Jacobian of the probability transformation from the original space \( \mathbf{x} \) to the standard normal space with respect to the parameters \( \mathbf{z'} \) and computed at the most likely failure point (design point) \( \mathbf{u'} \); \( \mathbf{z'} = \) design point for \( \mathbf{z} = (\mathbf{r}, \mathbf{\Theta}, \mathbf{\epsilon}_0) \), and \( \mathbf{SD}' \) = the SD diagonal matrix of equivalent normal variables \( \mathbf{z}' \), defined by the linearized inverse transformation \( \mathbf{z}' = \mathbf{z'} + \mathbf{J}_{\mathbf{x}}^{-1} \cdot (\mathbf{u}' - \mathbf{u}) \) at the design point. Each element in \( \mathbf{SD}' \) is the square root of the corresponding diagonal element of the covariance matrix \( \mathbf{\Sigma}' = \mathbf{J}_{\mathbf{x}}^{-1} \cdot \mathbf{J}_{\mathbf{x}}^{-1} \) of the variables in \( \mathbf{z}' \).

Fig. 10 shows the importance measures of the random variables as a function of time for 16-mm (5/8 in.) bars. Based on the formulation of \( \mathbf{\delta} \) and \( \mathbf{\gamma} \), the interpretation of the signs of the sensitivity and importance analysis are opposite of each other. Therefore, the results shown in Fig. 10 are consistent with those shown in Fig. 9. In particular, the importance measure for \( \mathbf{\epsilon}_0 \) is negative, indicating that \( \mathbf{\epsilon}_0 \) serves as a capacity variable (as already noted in Fig. 9). Similarly, the importance measure for \( \mathbf{\epsilon} \) is positive, indicating that \( \mathbf{\epsilon} \) serves as a demand variable because it controls the reduction in capacity over time. The random errors \( \mathbf{\epsilon}_0 \) and \( \mathbf{\epsilon} \) are the most important capacity and demand random variables, respectively. Also, the importance measure of \( \mathbf{\alpha} \) increases rapidly and consistently with time as with the formulation in Eq. (11). Similar observations are valid for 10- and 19-mm (3/8 and 3/4 in.) bars.
Approximate Point Estimates

Based on the results of the importance analysis, a second approximate solution is obtained by considering only the uncertainties from the most important random variables and disregarding the uncertainties in the less important ones. Because the uncertainty in $e_0$ and $e$ prevails over the other sources, a point estimate of Eq. (8) is obtained using point estimates (e.g., the nominal values or the means) $\hat{f}_{fla}$, $\hat{x}$, and $\hat{\Theta}$ in place of $f_{fla}$, $x$, and $\Theta$ as follows:

$$P(\hat{f}_{fla}, \hat{x}, \hat{\Theta}) = P\left[g(\hat{f}_{fla}, \hat{x}, \hat{\Theta}) \leq 0\right]$$

$$= \Phi \left( \frac{\hat{f}_{fla} - \mu_{\sigma_0}}{\mu_{\sigma_n}} \right) \left( 1 - \lambda \left[ \frac{D_{T, ref} \cdot \sigma_0}{E_r} \left( \frac{1}{T_{ref} - 1/T} \right) \right]^{2a} \right)$$

$$\left( \frac{1}{\sigma_0 + \lambda^2 \left[ \frac{D_{T, ref} \cdot \sigma_0}{E_r} \left( \frac{1}{T_{ref} - 1/T} \right) \right]^{2a}} \right)^{2a}$$

(25)

For the purpose of the analysis conducted in this section, the ACI requirement, $\hat{f}_{fla}$, was computed again using $\sigma/\mu_{\sigma_n} = 0.05$. Furthermore, the reliability index (Ditlevsen and Madsen 1996) corresponding to the probability in Eq. (9) is obtained as

$$\hat{\beta}(\hat{f}_{fla}, \hat{x}, \hat{\Theta}) = \Phi^{-1} \left[ 1 - P(\hat{f}_{fla}, \hat{x}, \hat{\Theta}) \right]$$

(26)

Fig. 8 shows $P$ for 16-mm (5/8 in.) bars (dotted line) as a function of time, compared with $P$ estimated using MC simulations, FORM, and $P_1$. It can be seen that $P$ tends to slightly overestimate the actual probability of being below the ACI specifications as time increases. As a value of reference, $P$ is higher than the estimate based on MC simulations by about 5% after approximately 50 years of exposure. Because neither $P$ or $P_1$ require specialized reliability software, both are suggested for future computations. In particular, $P$ is suggested to estimate the probability for short duration of exposure and $P_1$ is suggested to estimate the probability for long duration of exposure. Similar considerations are also valid for 10- and 19-mm (3/8 and 3/4 in.) bars.

Conclusions

GFRP reinforcing bars can provide many advantages to the owners and constructors of infrastructure systems. Although the advantages are many, the acceptance of using GFRP bars has been hampered by the lack of longer term data on residual strengths and the lack of probabilistic models for bars when embedded in concrete.

This paper developed a state-of-the-art model to predict the performance of GFRP bars embedded in concrete using three- and seven-year data on GFRP embedded in concrete, capturing the dependency of the tensile strength on time and the initial bar size. The developed probabilistic model is unbiased and properly accounts for the relevant sources of uncertainties, including the statistical uncertainty in the estimation of the unknown model parameters and the model error associated with the inexact model form.

The model indicates that the decay of the mean tensile strength is rapid over the first few years and gradually slows as time increases. Furthermore, the decay is more pronounced for smaller bars than for larger bars. The developed probabilistic model is also used to assess the probability that the actual tensile strength of GFRP bars does not meet the ACI 440 minimum capacity requirement over longer time. The model predicts that for a specified bar size, the probability of not meeting the design requirements increases with time. Conversely, at the same time, the probability decreases as the bar size increases. In particular, in 75 years and at an exposure temperature of 23°C (73°F), 10-mm (3/8 in.) bars reach a 0.37 probability of not meeting the ACI 440 requirement, 16-mm (5/8 in.) bars reach a 0.23 probability, and 19-mm (3/4 in.) bars reach a 0.18 probability. It is also shown that these probabilities increase with an increase of the exposure temperature.

Although the developed model provides valuable information on the long-term performance of GFRP bars embedded in concrete, additional research is needed to assess the time-variant structural reliability of whole structures (decks, pavements, and other infrastructure elements) over time. A reliability analysis would answer the fundamental question on the actual safety of structures with GFRP bars. The developed probabilistic model could be used to assess the structural capacity over time for a time-variant structural reliability analysis. It should also be noted that the GFRP reinforcing bars assessed in this research were embedded in concrete that was not subjected to loads other than the self-weight of the beam. The literature indicates that the residual capacity of GFRP subjected to load is less than GFRP bars subjected to no load.

Further research is needed to determine how these reduced tensile strengths can influence the performance of GFRP reinforced structures. However, this research indicates that the reduction in tensile capacity of GFRP reinforcing bars embedded in concrete is a function of bar diameter, diffusion characteristic of the GFRP polymer resin, and time. Using larger diameters of GFRP bars and lower design tensile strengths may extend the anticipated service life of GFRP reinforced structures. However, further research on the performance of GFRP RC specimens that have been exposed to loads for longer durations is recommended.

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The following symbols are used in this paper:

\( A \) = frequency factor;
\( C \) = alkaline concentration in percent;
\( C_E \) = environmental reduction factor;
\( D \) = measured data;
\( D_T \) = diffusion coefficient of the GFRP polymer matrix at temperature \( T \);
\( E(\cdot) \) = expectation function;
\( E_a \) = activation energy;
\( f_{fu} \) = design tensile strength of GFRP reinforcing bars;
\( \hat{f}_{fu} \) = point estimate of \( f_{fu} \);
\( f^{\ast}_{fu} \) = guaranteed ultimate tensile strength (GUTS) of a GFRP bar;
\( f_{u,ave} \) = mean tensile strength of a set of test specimens;
\( f(x, \Theta_f) \) = probability density function of \( x \) with distribution parameters \( \Theta_f \);
\( f(\Theta) \) = posterior probability density function of \( \Theta \);
\( g(\cdot) \) = limit state function;
\( g_1(y) \) = first-order approximation of \( g(y) \);
\( J_{w',x'} \) = Jacobian of the probability transformation from the original space \( x \) to the standard normal space with respect to the parameters \( z' \) and computed at the most likely failure point (design point) \( u^* \);
\( k \) = specified number of SDs;
\( k_r \) = rate constant;
\( L(\Theta) \) = likelihood function;
\( M_\Theta \) = posterior mean vector of \( \Theta \);
\( P \) = expected value of \( P(\Theta) \) over the distribution of \( \Theta, f(\Theta) \);
\( \hat{P} \) = point estimate of \( P(\Theta) \);
\( P(\cdot|\cdot) \) = conditional probability;
\( \hat{P}_1 \) = first-order approximation of \( \hat{P} \);
\( P(\Theta) \) = probability of not meeting the design specifications;
\( p(\Theta) \) = previous distribution of \( \Theta \);
\( R \) = universal gas constant;
\( r \) = GFRP bar radius;
\( \hat{r} \) = point estimate (i.e., mean) of \( r \);
\( \text{SD'} \) = SD diagonal matrix of equivalent normal variables \( x' \);
\( s \) and \( s_0 \) = SDs of two error terms;
\( s_{fi} \) = SD of \( g_1(y) \);
\( T \) = ambient exposure temperatures;
\( T_{ref} \) = ambient reference exposure temperatures;
\( t \) = embedment time;
\( u^* \) = most likely failure point (design point);
\( X \) = alkali penetration depth;
\( x = (D_{T,ref}, E_{un}, R, r, T) \) = vector of basic variables;
\( \hat{x} \) = point estimate of \( x \);
\( y = (f_{fu}, x, \Theta) \) = vector of random variables;
\( \hat{y} \) = point estimate (i.e., mean) of \( y \);
\( z' \) = design point for \( z = (r, \Theta, \varepsilon_c) \);
\( z' \) = equivalent normal variables;
\( \alpha \) = model parameter;
\( \beta \) = generalized reliability index corresponding to \( P \);
\( \beta \) = generalized reliability index corresponding to \( P \);
\( \gamma \) = vector of importance measures;
\( \delta \) = vector of sensitivity measures;
\( \varepsilon_0 \) and \( \varepsilon \) = standard normal random variables;
\( \Theta = (\lambda, \alpha, s_0, \delta) \) = vector of unknown model parameters;
\( \hat{\Theta} \) = point estimate of \( \Theta \);
\( \Theta_f \) = vector of distribution parameters;
\( \lambda \) = model parameter;
\( \mu_{s_0} \) = mean of \( s_0 \);
\( \mu_{\sigma_0} \) = mean of \( \sigma_0 \);
\( \rho_i \) = difference between the \( i \)th measured and the corresponding predicted normalize stresses;
\( \sigma \) = SD of the tensile strength of a set of test specimens;
\( \sigma_1 \) = tensile strength after exposure (stress units);
\( \sigma_0 \) = tensile strength before exposure (stress units);
\( \Sigma \) = diagonal matrix with diagonal elements given by the SD of each random variable in \( x \);
\( \Sigma_y \) = covariance matrix of \( y \);
\( \Sigma_{yy} \) = covariance matrix of the variables \( z' \);
\( \Sigma_{\Theta_0} \) = posterior covariance matrix of \( \Theta \);
\( \varphi(\cdot) \) = standard normal probability density function;
\( \Phi(\cdot) \) = standard normal cumulative distribution function;
\( \Phi^{-1}(\cdot) \) = inverse of the standard normal cumulative distribution function;
\( \psi \) = normalizing factor; and
\( \nabla_{\Theta, \beta} \) = gradient vector of \( \beta \) with respect to \( \Theta_f \) computed at the mean point.

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